BCS-BEC crossover in a gas of Fermi atoms with a p-wave Feshbach resonance

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Abstract

We investigate unconventional superfluidity in a gas of Fermi atoms with an anisotropic p-wave Feshbach resonance. Including the p-wave Feshbach resonance as well as the associated three kinds of quasi-molecules with finite orbital angular momenta $L_z = \pm 1, 0$, we calculate the transition temperature of the superfluid phase. As one passes through the p-wave Feshbach resonance, we find the usual BCS-BEC crossover phenomenon. The p-wave BCS state continuously changes into the BEC of bound molecules with $L = 1$. Our calculation includes the effect of fluctuations associated with Cooper-pairs and molecules which are not Bose-condensed.

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The search for p-wave superfluidity is the next big challenge in a trapped Fermi gas, after the discovery of s-wave superfluidity in ^{40}K and ^{6}Li[1, 2, 3, 4, 5]. Recently, p-wave Feshbach resonances have been observed[6, 7, 8]. The discovery of p-wave superfluidity will be the first realization of pseudo-spin *triplet* superfluidity in a Fermi atomic gas, quite different from the recently discovered *singlet* s-wave superfluidity[1, 2, 3, 4, 5]. Since the pairing interaction associated with a Feshbach resonance can be tuned by varying the threshold energy 2ν of the resonance (see below), one can probe the p-wave BCS-BEC crossover. The superfluidity will continuously change from p-wave BCS-type to a BEC-type of bound molecules with a finite angular momentum $L = 1$, as one passes through the Feshbach resonance.

As a useful first step, we calculate the superfluid phase transition temperature $T_{\rm c}$ over the entire p-wave BCS-BEC crossover regime. We explicitly include the p-wave Feshbach resonance and associated molecules with three values of $L_z = \pm 1, 0$. To describe the BCS-BEC crossover[9, 10], it is necessary to include the fluctuations in the three p-wave Cooper-channels and their coupling due to the strong pairing interaction associated with the Feshbach resonance. Our work is a generalization of Ref. [10]. We consider both a single-component Fermi gas (where a Feshbach resonance occurs in the same hyperfine state) as well as a two-component Fermi gas (where a Feshbach resonance occurs between different hyperfine states). The Feshbach resonances in both cases have been recently observed[6, 7, 8]. We deal with both a narrow and a broad Feshbach resonance. Although we mainly consider a uniform gas in this letter, we discuss $T_{\rm c}$ in a trapped gas in the BEC limit.

p-wave superfluidity in trapped Fermi gases was discussed in the BCS regime at $T_{\rm c}$[11, 12]. Very recently, the p-wave gap equation for the order parameter of the superfluid phase in the crossover region was solved in Ref. [13] at $T = 0$. An attractive interaction in the $L \neq 0$ partial wave channel was considered. An interesting phase transition in a two-dimensional Fermi gas has also been predicted[14]. In contrast to these recent papers, our starting point explicitly introduces the molecules which form as a result of the Feshbach resonance. We thus emphasize the physical nature of the p-wave bound states which form the Bose condensate in the crossover region. We also remark that a p-wave pairing mechanism has been proposed using the dipole interaction[15].

We extend the coupled fermion-boson (CFB) model for a s-wave Feshbach resonance[10,

16, 17] to a p-wave one,

$$
\begin{aligned}
H &= \sum_{\mathbf{p}} \varepsilon_{\mathbf{p}} c^{\dagger}_{\mathbf{p}} c_{\mathbf{p}} + \sum_{\mathbf{q},j} [\varepsilon^{B}_{\mathbf{q}} + 2\nu] b^{\dagger}_{\mathbf{q},j} b_{\mathbf{q},j} \\
&- \frac{U}{2} \sum_{\mathbf{p},\mathbf{p}',\mathbf{q},j} \mathbf{p}\cdot\mathbf{p}' c^{\dagger}_{\mathbf{p}+\mathbf{q}/2} c^{\dagger}_{-\mathbf{p}+\mathbf{q}/2} c_{-\mathbf{p}'+\mathbf{q}/2} c_{\mathbf{p}'+\mathbf{q}/2} \\
&+ \frac{g_{\rm r}}{\sqrt{2}} \sum_{\mathbf{p},\mathbf{q},j} p_j \Big[b_{\mathbf{q},j} c^{\dagger}_{\mathbf{p}+\mathbf{q}/2} c^{\dagger}_{-\mathbf{p}+\mathbf{q}/2} + h.c. \Big]. \qquad (1)
\end{aligned}
$$

Here $c^{\dagger}_{\mathbf{p}}$ is a creation operator of a Fermi atom with the kinetic energy $\varepsilon_{\mathbf{p}} \equiv p^2/2m$. $b_{\mathbf{q},j}$ describe three kinds of molecular bosons (labelled by $j = x, y, z$), all with the center of mass momentum $\mathbf{q}$, associated with the p-wave Feshbach resonance. The threshold energy 2ν in the molecular kinetic energy $\varepsilon^{B}_{\mathbf{q}} + 2\nu \equiv q/2M + 2\nu$ is independent of j, due to the spherical symmetry of the system we are considering. In the last term, $g_{\rm r}$ is the coupling constant of a p-wave Feshbach resonance, with p_j characterizing the p-wave symmetry[18].

The Feshbach resonance term in (1) is obtained from a more general Hamiltonian $H_{\rm F.R.} \equiv \int d\mathbf{r} d\mathbf{r}' [g_{\rm r}(\mathbf{r}-\mathbf{r}')\Phi(\mathbf{r},\mathbf{r}')\Psi^{\dagger}(\mathbf{r})\Psi^{\dagger}(\mathbf{r}') + h.c.]$. Here $\Psi(\mathbf{r}) \equiv \sum_{\mathbf{p}} e^{i\mathbf{p}\cdot\mathbf{r}} c_{\mathbf{p}}$ is a fermion field operator, and

$$
\Phi(\mathbf{r},\mathbf{r}') \equiv \sum_{\mathbf{q}} e^{i\mathbf{q}\cdot\mathbf{R}} \sum_{n,L,L_z} u_{nL}(\tilde{r}) Y_{L,L_z}(\theta,\phi) b_{\mathbf{q},n,L,L_z} \qquad (2)
$$

describes molecules with center of mass $\mathbf{R} \equiv (\mathbf{r}+\mathbf{r}')/2$, and relative coordinate $\tilde{\mathbf{r}} \equiv \mathbf{r}-\mathbf{r}'$. $b_{\mathbf{q},n,L,L_z}$ is an annihilation operator of a bound molecular state, described by the eigenfunction $u_{nL}(\tilde{r})Y_{L,L_z}(\theta,\phi)$. The last term in (1) is obtained when we retain the terms in $\Psi(\mathbf{r})$ to leading order in $\mathbf{p}$, in the $L = 1$ channel ($b_{\mathbf{q}L_z} \equiv b_{\mathbf{q},n,L=1,L_z}$) for a Feshbach resonance state specified by a radial quantum number n. We note that $p_x \propto Y_{11} + Y_{1,-1}$, $p_y \propto Y_{11} - Y_{1,-1}$ and $p_z \propto Y_{1,0}$. Thus the molecular operators $b_{\mathbf{q},j}$ in (1) are related to $b_{\mathbf{q},L_z}$, with azimuthal angular momentum components $L_z = \pm 1, 0$, as follows $(b_{\mathbf{q},x}, b_{\mathbf{q},y}, b_{\mathbf{q},z}) = (\frac{1}{\sqrt{2}}[b_{\mathbf{q},1} + b_{\mathbf{q},-1}], \frac{i}{\sqrt{2}}[b_{\mathbf{q},1} - b_{\mathbf{q},-1}], b_{\mathbf{q},0})$. Equation (1) also includes a non-resonant p-wave interaction U[18], which we take to be attractive ($-U < 0$).

In the p-wave Feshbach resonance, since two Fermi atoms form one of the three kinds of quasi-molecular bosons described by $b^{\dagger}_{\mathbf{q},j}$ ($j = x, y, z$) and this bound state can dissociate into two Fermi atoms, we take $M = 2m$ and impose the conservation of the total number of Fermi atoms as $N = N_{\rm F} + 2\sum_{j=x,y,z} N^{j}_{\rm B}$. Here $N_{\rm F}$ is the number of Fermi atoms and $N^{j}_{\rm B}$ is the number of Bose molecules in the j-th component. This constraint can be actually absorbed into (1) by considering the grand-canonical Hamiltonian $H \equiv H - \mu N$. The resulting

Hamiltonian has the same form as (1), where $\varepsilon_{\mathbf{p}}$ and $\varepsilon^B_{\mathbf{q}}$ are replaced by $\xi_{\mathbf{p}} \equiv \varepsilon_{\rm p} - \mu$ and $\xi^B_{\mathbf{q}} \equiv \varepsilon^B_{\mathbf{q}} - 2\mu$, respectively.

The superfluid phase is characterized by three anisotropic p-wave Cooper-pairs $\Delta_j(\mathbf{p}) \equiv U \sum_{\mathbf{p}'} p_j p'_j \langle c_{-\mathbf{p}'} c_{\mathbf{p}'} \rangle$ and three molecular BEC order parameters $\phi_j \equiv \langle b_{\mathbf{q}=0,j} \rangle$ ($j = x, y, z$). In the equilibrium state, these are related to each other through the identity[10] $p_j \phi_j = -\frac{g_{\rm r}}{\sqrt{2}U} \frac{1}{2\nu - 2\mu} \Delta_j(\mathbf{p})$. The single-particle excitations have the BCS spectrum $E_{\mathbf{p}} = \sqrt{\xi^2_{\mathbf{p}} + |\sum_j \tilde{\Delta}_j(\mathbf{p})|^2}$ with the composite order parameter, given by $\tilde{\Delta}_j(\mathbf{p}) \equiv \Delta_j(\mathbf{p}) - \sqrt{2} g_{\rm r} p_j \phi_j$. The angular dependence of $\tilde{\Delta}_j(\mathbf{p})$ is proportional to p_j. This composite order parameter is self-consistently determined by the BCS gap equation, $1 = \frac{1}{3} U_{\rm eff} \sum_{\mathbf{p}} \frac{p^2}{2E_{\rm p}} \tanh \frac{E_{\mathbf{p}}}{2T}$, where the factor $p^2/3$ comes from the angular integration of p_j^2. The effective pairing interaction $U_{\rm eff} \equiv U + g_{\rm r}^2/(2\nu - 2\mu)$ includes the effect of Feshbach resonance[10]. In the weak-coupling or BCS regime, $\mu \simeq \varepsilon_{\rm F}$ (where $\varepsilon_{\rm F}$ is the Fermi energy).

The analogous p-wave CFB model for a two-component Fermi gas ($\equiv \uparrow, \downarrow$) is described by

$$\begin{aligned} H \;=\; & \sum_{\mathbf{p},\sigma} \varepsilon_{\mathbf{p}} c^\dagger_{\mathbf{p}\sigma} c_{\mathbf{p}\sigma} + \sum_{\mathbf{q},j} [\varepsilon^B_{\mathbf{q}} + 2\nu] b^\dagger_{\mathbf{q},j} b_{\mathbf{q},j} \\ & - \; U \sum_{\mathbf{p},\mathbf{p}',\mathbf{q},j} \mathbf{p} \cdot \mathbf{p}' c^\dagger_{\mathbf{p}+\mathbf{q}/2\uparrow} c^\dagger_{-\mathbf{p}+\mathbf{q}/2\downarrow} c_{-\mathbf{p}'+\mathbf{q}/2\downarrow} c_{\mathbf{p}'+\mathbf{q}/2\uparrow} \\ & + \; g_{\rm r} \sum_{\mathbf{p},\mathbf{q},j} p_j \Big[b_{\mathbf{q},j} c^\dagger_{\mathbf{p}+\mathbf{q}/2\uparrow} c^\dagger_{-\mathbf{p}+\mathbf{q}/2\downarrow} + h.c. \Big]. \end{aligned} \tag{3}$$

In a mean field pairing approximation, we again obtain the same single-particle excitations $E_{\mathbf{p}}$ and the gap equation as those in the single-component case. The Cooper-pair order parameter is $\Delta_j(\mathbf{p}) \equiv U \sum_{\mathbf{p}'} p_j p'_j \langle c_{-\mathbf{p}'\downarrow} c_{\mathbf{p}'\uparrow} \rangle$.

We now present the p-wave strong-coupling theory at $T_{\rm c}$ for the single-component (spin polarized) model defined in (1). The discussion is easily extended to the two-component case. The equation for $T_{\rm c}$ is obtained by employing the Thouless criterion[9, 10], the temperature when the particle-particle scattering matrix first develops a pole at $\omega = \mathbf{q} = 0$. In the t-matrix approximation, the p-wave scattering matrix has the form $\tilde{\Gamma}_{ij}(\mathbf{p}, \mathbf{p}', \mathbf{q}, \omega) = p_i \Gamma_{ij}(\mathbf{q}, \omega) p_j$, which is shown diagrammatically in Fig. 1(a). In this figure, the first and the second lines, respectively, describe the effects of non-resonant interaction U and the p-wave Feshbach resonance, that give $\hat{\Gamma}(\mathbf{q}, \omega) \equiv \{\Gamma_{ij}\} = -[1 - U_{\rm eff}(\mathbf{q}, \omega) \hat{\Pi}(\mathbf{q}, \omega)]^{-1} U_{\rm eff}(\mathbf{q}, \omega)$ ($i, j = x, y, z$). Here, $U_{\rm eff}(\mathbf{q}, \omega) \equiv U - g_{\rm r}^2 D_0(\mathbf{q}, \omega)$ is an atom-atom interaction including dynamical effects described by the bare molecular Bose propagator $D_0^{-1}(\mathbf{q}, \omega) \equiv \omega + i\delta - [\xi^B_{\mathbf{q}} + 2\nu]$. The correlation functions $\hat{\Pi} \equiv \{\Pi_{ij}\}$ are obtained from the analytic continuation $i\nu_n \to \omega + i\delta$

of the two-particle thermal Green's function,

$$\Pi_{ij}(\mathbf{q}, i\nu_n) \equiv \frac{1}{\beta}\sum_{\mathbf{p}} p_i p_j \frac{1 - f(\xi_{\mathbf{p}+\mathbf{q}/2}) - f(\xi_{\mathbf{p}-\mathbf{q}/2})}{\xi_{\mathbf{p}+\mathbf{q}/2} + \xi_{\mathbf{p}-\mathbf{q}/2} - i\nu_n}, \tag{4}$$

where $f(\varepsilon)$ is the Fermi distribution function. The diagonal components Π_{ii} $(i = x, y, z)$ describe superfluid fluctuations in the i-th Cooper-channel, while the off-diagonal components give the coupling of fluctuations in *different* channels. Noting that $\Pi_{i\neq j}(0, 0) = 0$ in our approximation, the Thouless criterion gives the equation for $T_{\rm c}$ as

$$1 = U_{\rm eff}\Pi_{ii}(0, 0) = \frac{1}{3}U_{\rm eff}\sum_{\mathbf{p}} \frac{p^2}{2(\varepsilon_{\mathbf{p}} - \mu)} \tanh\frac{\varepsilon_{\mathbf{p}} - \mu}{2T}. \tag{5}$$

This has the same form as the mean-field gap equation at $T = T_{\rm c}$, with $\tilde{\Delta}_j(\mathbf{p}) \to 0$. However, the chemical potential μ in (5) can be quite different from the Fermi energy $\varepsilon_{\rm F}$ in the crossover regime[9, 10, 19, 20], and one needs an additional equation to determine μ.

The chemical potential μ is determined from the equation for the number of atoms N, which is calculated from the thermodynamic potential Ω using the formula $N = -\frac{\partial\Omega}{\partial\mu}$[9, 10]. Figure 1(b) shows the fluctuation correction to Ω, where the diagrams on the left and right describe fluctuations in the p-wave Cooper-channels and the Feshbach resonance, respectively. Summing up these diagrams, we obtain

$$\begin{aligned} N &= N_{\rm F} - \frac{1}{\beta}\sum_{\mathbf{q}, i\nu_n} \frac{\partial}{\partial\mu} tr\Big[\log\hat{D}(\mathbf{q}, i\nu_n)\Big] \\ &- \frac{1}{\beta}\sum_{\mathbf{q}, i\nu_n} \frac{\partial}{\partial\mu} tr\Big[\log[1 - U\hat{\Pi}(\mathbf{q}, i\nu_n)]\Big] \\ &\equiv N_{\rm F} + 2N_{\rm B} + 2N_{\rm C}, \end{aligned} \tag{6}$$

where the trace is taken over the $L = 1$ space $(j = x, y, z)$. $N_{\rm F} \equiv \sum_{\mathbf{p}} f(\xi_{\mathbf{p}})$ is the number of free Fermi atoms. $N_{\rm B}$ is the number of Feshbach molecules, given as the poles of the renormalized (matrix) molecular Bose Green's function $\hat{D}^{-1}(\mathbf{q}, i\nu_n) \equiv i\nu_n - (\xi^B_{\mathbf{q}} + 2\nu) - \hat{\Sigma}(\mathbf{q})$. The molecular self-energy $\hat{\Sigma}(\mathbf{q}, i\nu_n) \equiv -g_{\rm r}^2\hat{\Pi}/(1 - U\hat{\Pi})$ describes the fluctuation effects in the p-wave Cooper-channels. As in Refs. [9, 10], $N_{\rm C}$ can be interpreted as the contribution of preformed p-wave Cooper-pairs as well as particle-particle scattering states. We note that superfluid fluctuations in the three p-wave Cooper-channels are strongly coupled to one another in (6) through Π_{ij} $(i \neq j)$. Equations (5) and (6) are the basic coupled equations describing $T_{\rm c}$ of a uniform p-wave superfluid over the entire BCS-BEC crossover.

The same equations are obtained in the two-component case in (3), with $N_{\rm F}$ replaced by $N_{\rm F} = 2\sum_{\mathbf{p}} f(\xi_{\mathbf{p}})$, reflecting the two Fermi hyperfine states.

Figure 2 shows the self-consistent numerical solutions of (5) and (6). In this figure, we have introduced a p-wave scattering length a_p for the renormalized interaction $U_{\rm eff}^R$ which occurs in the gap equation when written in a cutoff-independent way[21]. This is defined[11, 12] by $-4\pi(3a_p^3)/m \equiv U_{\rm eff}^R = U_{\rm eff}/(1 - \frac{U_{\rm eff}}{3}\sum_{[0,\omega_c]} \frac{p^2}{2\varepsilon_{\mathbf{p}}})$, where ω_c is a high-energy cutoff. The increase of $(k_{\rm F}a_p)^{-3}$ corresponds to a decrease of bare threshold energy 2ν. Since the chemical potential also decreases to approach ν [see the inset in panel 2(b)], the bare interaction $U_{\rm eff} = U + g_{\rm r}^2/(2\nu - 2\mu)$ becomes stronger for larger $(k_{\rm F}a_p)^{-3}$. In the BCS regime $[(k_{\rm F}a_p)^{-3} \lesssim -1]$, $T_{\rm c}$ agrees well with the standard weak-coupling BCS theory ['BCS' in Fig. 2(a)]. On the other hand, in the crossover regime $[-1 \lesssim (k_{\rm F}a_p)^{-3} \lesssim 0]$, the deviation of $T_{\rm c}$ from the weak-coupling result is large. The chemical potential μ also begins to strongly deviate from the Fermi energy $\varepsilon_{\rm F}$, as shown in Fig. 2(b). Figure 3 shows that the gas continuously changes from a gas of Fermi atoms (dominated by $N_{\rm F}$) into a Bose gas of bound states (dominated by $N_{\rm M} = N_{\rm B} + N_{\rm C}$). In the BEC regime $[(k_{\rm F}a_p)^{-3} \lesssim 0]$, free Fermi atoms are almost absent, and $T_{\rm c}$ approaches a constant value. Its precise value depending on whether one is dealing with a single-component gas ($\uparrow\uparrow$) or a two-component gas ($\uparrow\downarrow$). This difference is due to different Fermi energies in the two cases (see TABLE I). The peak in $T_{\rm c}$ in Fig. 2(a) would be absent if the coupling to the bound states[10, 22] was properly included in the self-energies of the Fermi atoms.

In the extreme BEC limit, where all the atoms have formed Feshbach molecules ($N_{\rm F},\ N_{\rm C} = 0$), the gas can be regarded as a non-interacting Bose gas mixture with three kinds of Feshbach molecules, with $L_z = \pm 1, 0$. In this case, rewriting Eq. (5) as $2\mu = 2\nu - g_{\rm r}^2\Pi_{ii}(0,0)/[1 - U\Pi_{ii}(0,0)]$, we find $2\mu \to 2\nu$, because $\Pi_{ii}(0,0) = 0$ in this BEC limit. This result is consistent with the inset in Fig. 2(b). Since 2μ is the chemical potential of the molecular Bose gas and 2ν is the threshold energy of molecular excitations, the condition $2\mu = 2\nu$ is that required for BEC in a non-interacting Bose gas. That is to say, $T_{\rm c}$ in this extreme case is simply determined by $N = 3\sum_{\mathbf{q}} n_B(\varepsilon_{\mathbf{q}}^B)$, where $n_B(\varepsilon)$ is the Bose distribution function. The factor 3 comes from the presence of *three* kinds of molecules, which is characteristic of p-wave superfluidity. Because of this factor, $T_{\rm c}$ in the p-wave case is lower than the s-wave case, as shown in TABLE I. TABLE I also shows $T_{\rm c}$ in the BEC limit in a trapped gas, evaluated within the LDA. These values for $T_{\rm c}$ in a trapped gas

seems accessible in current experiments. The crossover behavior of $T_{\rm c}$ shown in Fig. 2(a) is a general result valid for any type of p-wave superfluidity.

Figures 2 and 3 indicate that the crossover behavior of $T_{\rm c}$, μ, and the number of Fermi atoms $N_{\rm F}$ and that of Bose molecules $N_{\rm M}$ show quasi-universal behavior when plotted as a function of $(k_{\rm F}a_s)^{-3}$, irrespective of whether the Feshbach resonance is narrow ($\bar{g}_{\rm r} < \varepsilon_{\rm F}$) or broad ($\bar{g}_{\rm r} > \varepsilon_{\rm F}$). On the other hand, the character of the bound state bosons is different between the two. In a narrow Feshbach resonance, the Feshbach molecules ($N_{\rm B}$) are dominant in the crossover regime, while Cooper-pairs ($N_{\rm C}$) are dominant in a broad Feshbach resonance (see Fig. 3). However, Feshbach molecules always dominate in the extreme BEC limit.

We note that the phase diagrams in trapped Fermi gases[1, 2, 3, 4, 5] which experiments measure involve passing through the resonance in an adiabatic (constant entropy) manner. In the case of a s-wave Feshbach resonance, Ref. [23] has discussed how one can calculate such phase diagrams using a simple ideal gas model. This could be extended to the p-wave Feshbach resonance case.

To summarize, we have discussed the BCS-BEC crossover in the presence of a p-wave Feshbach resonance. Generalizing earlier work on the s-wave BCS-BEC crossover[10], we have included fluctuation effects in the three p-wave Cooper-channels, as well as the three kinds of Feshbach molecules with $L_z = \pm 1, 0$. Observation of the molecular condensate in the BEC regime[1, 2, 3, 4, 5] would be a first step in the study of p-wave superfluidity.

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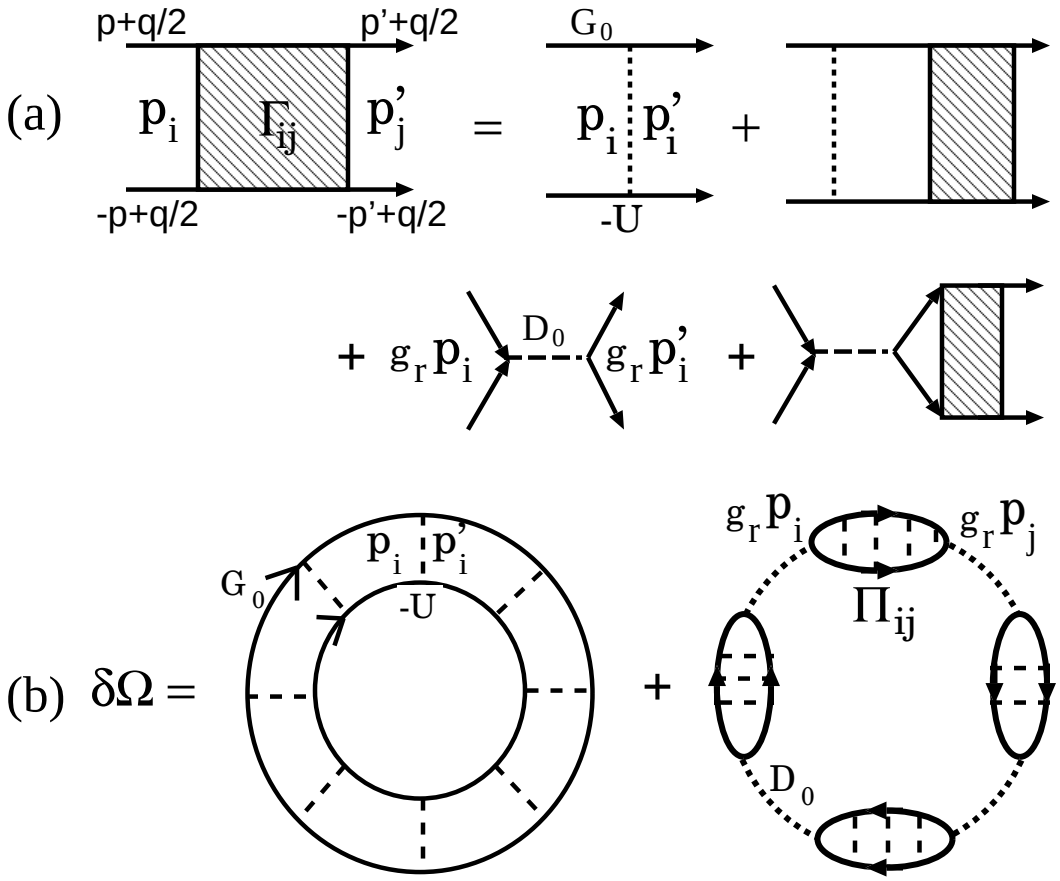
FIG. 1: (a) Particle-particle scattering matrix in the t-matrix approximation in terms of the non-resonant interaction U (first line) and the p-wave Feshbach resonance $g_{\rm r}$ (second line). G_0 and D_0 are the bare single-particle Fermi and Bose Green's function, respectively. (b) Corrections to the thermodynamic potential originating from fluctuations in the p-wave Cooper-channels (left diagram) and the Feshbach resonance (right diagram).

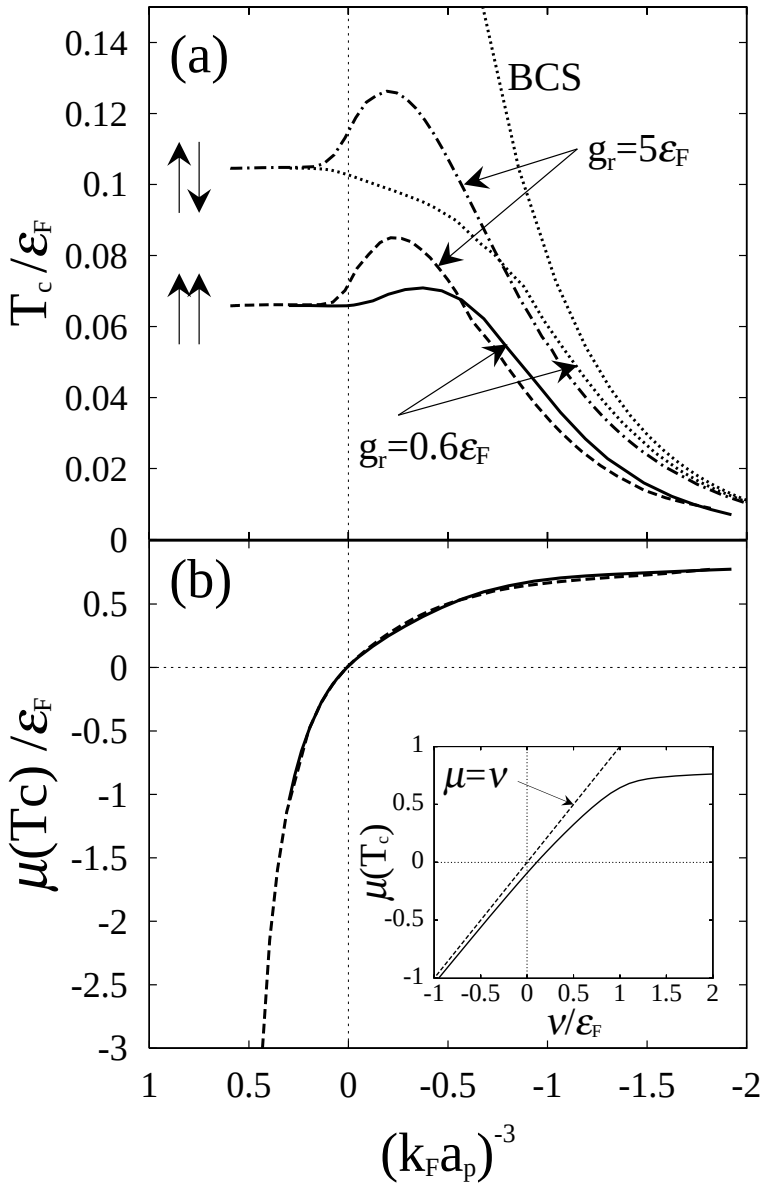
FIG. 2: (a) $T_{\rm c}$ in the p-wave BCS-BEC crossover. $\uparrow\uparrow$: single-component Fermi gas, with $\bar{U} \equiv Np_{\rm F}^2 U = 0.4\varepsilon_{\rm F}$. $\uparrow\downarrow$: two-component Fermi gas, with $\bar{U} = 0.8\varepsilon_{\rm F}$. Results for a narrow Feshbach resonance ($\bar{g}_{\rm r} \equiv \sqrt{Np_{\rm F}^2} g_{\rm r} = 0.6\varepsilon_{\rm F}$) and a broad Feshbach resonance ($\bar{g}_{\rm r} = 5\varepsilon_{\rm F}$) are shown. 'BCS' shows a weak-coupling result in the two-component case with μ being fixed at the value at $\nu = 2.5\varepsilon_{\rm F}$. (b) Chemical potential $\mu(T_{\rm c})$ in a single component Fermi gas. The solid and dashed lines show the results for a narrow and a broad Feshbach resonance, respectively. The inset shows $\mu(T_{\rm c})$ as a function of the threshold energy 2ν for a narrow Feshbach resonance.

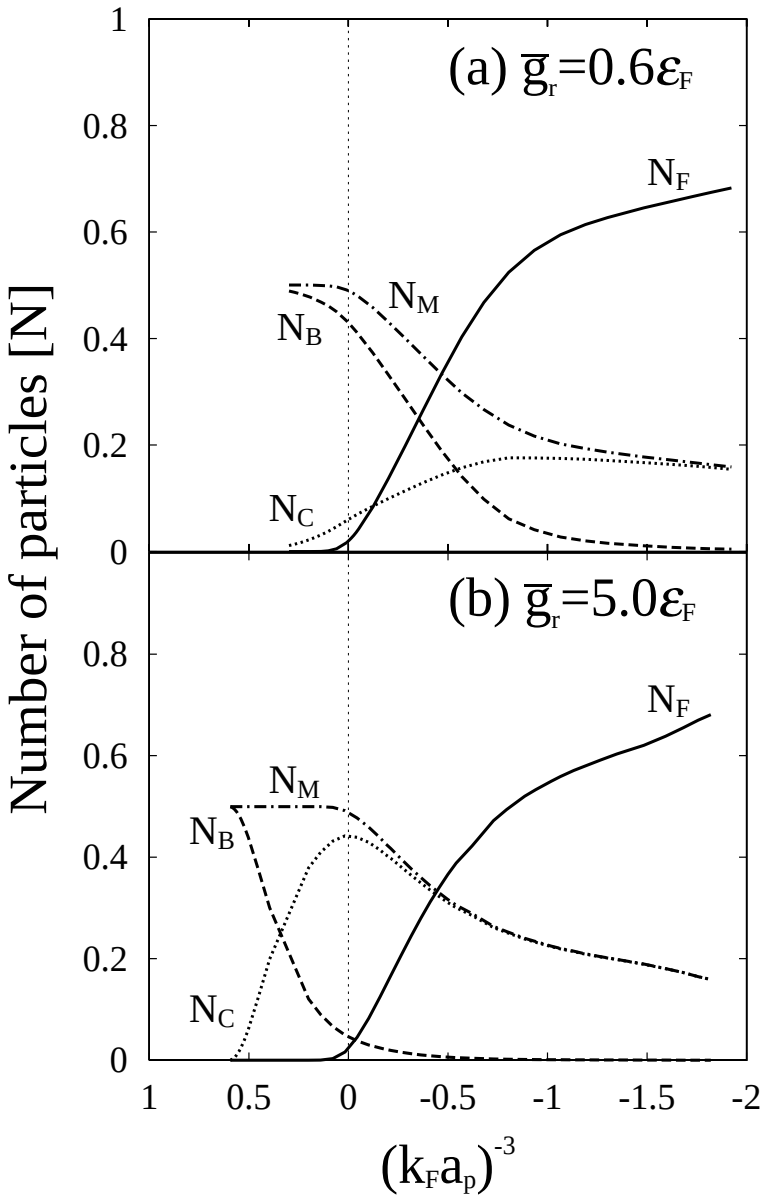
FIG. 3: Numbers for various kinds of particles at $T_{\rm c}$ in a single-component Fermi gas. (a) narrow Feshbach resonance, and (b) broad Feshbach resonance. $N_{\rm M} \equiv N_{\rm B} + N_{\rm C}$, where $N_{\rm B}$ describes Feshbach molecules and $N_{\rm C}$ gives the contribution from Cooper-pairs (stable and unstable).

TABLE I: $T_{\rm c}$ in the BEC limit. (S) and (T) show the single- and two-component Fermi gas, respectively. $T_{\rm c}$ in a uniform gas is given by $T_{\rm c} = \frac{2T_{\rm F}}{[6\alpha\sqrt{\pi}\zeta(3/2)]^{2/3}}$ (where $\zeta(z)$ is the zeta-function), with $\alpha = 1$ (s-wave), $\alpha = 6$ (S), and $\alpha = 3$ (T). In a harmonic trap, $T_{\rm c} = \frac{T_{\rm F}}{[6\alpha\zeta(3)]^{1/3}}$ is evaluated using the LDA. $T_{\rm F}$ ($= \varepsilon_{\rm F}$) is obtained from $N = \eta \sum_{\varepsilon \le \varepsilon_{\rm F}} 1$, where $\eta = 1$ ($\eta = 2$) for the single (two) component case.

symmetry	uniform gas [$T_{\rm F}$]	trapped gas [$T_{\rm F}$]
s-wave	0.218	0.518
p-wave (S)	0.066	0.285
p-wave (T)	0.105	0.359


(a)
p+q/2
p'+q/2
p_i
Γ_ij
p'_j
-p+q/2
-p'+q/2
=
G_0
p_i
p'_i
-U
+
+ g_r p_i
D_0
g_r p'_i
+
(b) δΩ =
G_0
p_i
p'_i
-U
+
g_r p_i
g_r p_j
Π_ij
D_0


(a)
BCS
$g_r=5\varepsilon_F$
$g_r=0.6\varepsilon_F$
T_c/ε_F
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
(b)
$\mu(Tc)/\varepsilon_F$
0.5
0
-0.5
-1
-1.5
-2
-2.5
-3
$\mu=\nu$
$\mu(T_c)$
ν/ε_F
1
0.5
0
-0.5
-1
1.5
2
$(k_F a_p)^{-3}$


(a) $\bar{g}_r=0.6\varepsilon_F$
(b) $\bar{g}_r=5.0\varepsilon_F$
N_F
N_M
N_B
N_C
Number of particles [N]
$(k_F a_p)^{-3}$
1
0.8
0.6
0.4
0.2
0
0.5
-0.5
-1
-1.5
-2